Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Directional Effects in 3-D Diffusers

HERMANN VIETS*
Aerospace Research Laboratories/LE,
Wright-Patterson Air Force Base, Ohio

I. Introduction

THE diffusion of flows which are not completely mixed (i.e., have a nonuniform velocity profile) is of interest in various fluid dynamic systems. This is particularly true of aircraft applications where the overall length of a system is of primary importance, thus leading to situations where unmixed flows must be diffused. Renewed interest has recently focused on this problem in relation to the development of a highly efficient thrust augmenting ejector.[1]

It is the purpose of this Note to examine the effect of direction on the three-dimensional diffusion process. Consider an unmixed velocity profile at the inlet to the diffuser. The question then arises: Is there a preferred direction of diffusion? That is, is it more effective to diffuse the flow in the plane of the velocity profile (Fig. 1a) or in the plane normal to that of the velocity profile (Fig. 1b)? The schematics in Fig. 1 show a square inlet in order to separate the directional effect from that of aspect ratio.

II. Analysis

For diffusers of moderate expansion angles, the character of the flow is that the cross flow velocities v and w are small compared to the streamwise velocity component u. Therefore, as in Refs. 2 and 3, the boundary-layer assumptions are applied in both the y and z directions. The resulting nondimensionalized boundary-layer equations are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{R}\left[\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right] \qquad (1a)$$

$$(\partial p/\partial y)=0 \qquad (1b)$$

$$(\partial p/\partial z)=0 \qquad (1c)$$

and continuity

$$\int_A u\,dA=\text{constant}=1 \qquad (1d)$$

The velocities are nondimensionalized with respect to the average velocity at the diffuser inlet, U_o, the distances with half the initial wall separation L_o and the pressures with the dynamic head ρU_o^2. Thus R is the Reynolds number, defined as $R=(\rho U_o L_o/\mu)$. In addition, due to the double symmetry about the y and z axes, the analysis will treat only one quadrant of the yz coordinate system. Therefore, the area $A(x)$ is one fourth the total area distribution.

Received April 9, 1974; revision received December 17, 1974; based on a paper presented at the 2nd International Symposium on Air-breathing Engines, Sheffield, England, March 25–29, 1974. The author wishes to acknowledge the constructive criticism supplied by Associate Editor N. H. Kemp, which greatly simplified the analysis. Thanks are due B. Quinn and D. A. Lee for several stimulating conversations and to V. Grycz for typing the manuscripts.

Index categories: VTOL Aircraft Design; Viscous Nonboundary Layer Flows.

* Aerospace Engineer. Member AIAA.

Now $U(x)$ is defined as the average nondimensional velocity in the diffuser duct. Then, from the conservation of mass

$$U(x)=[U(0)A(0)/A(x)]=[1/A(x)] \qquad (2)$$

where $A(x)=g(x)h(x)$ is the streamwise area distribution in one quadrant.

Consider the initial velocity profiles (at $x=0$), such that the deviation of the velocity from the average velocity $[u(o,y)-U_o]$ is small. If, in addition, the diffuser angles are small, the transverse velocities v and w are small. Then the streamwise momentum equation (1a) may be linearized about $U(x)$ by introducing $u'=u-U(x)$, noting that u', v, w, and $(\partial U/\partial x)$ are small and neglecting terms of higher than first order in these variables.

The resulting streamwise momentum equation is [including Eq. (2)]

$$\frac{1}{A(x)}\left[\frac{d}{dx}\left(\frac{1}{A(x)}\right)+\frac{\partial u'}{\partial x}\right]=-\frac{dp}{dx}+\frac{1}{R}\left[\frac{\partial^2 u'}{\partial y^2}+\frac{\partial^2 u'}{\partial z^2}\right] \qquad (3)$$

Also, because of the definition of u' as the deviation from the average velocity

$$\int_o^g\int_o^h u'\,dy\,dz=0 \qquad (4)$$

In neither diffuser flow situation is the velocity a function of z; so $\partial u'/\partial z=0$ and Eq. (3) becomes

$$\frac{1}{A}\frac{\partial u'}{\partial x}+\left[\frac{1}{A}\frac{d}{dx}\left(\frac{1}{A}\right)+\frac{dp}{dx}\right]=\frac{1}{R}\frac{\partial^2 u'}{\partial y^2} \qquad (5)$$

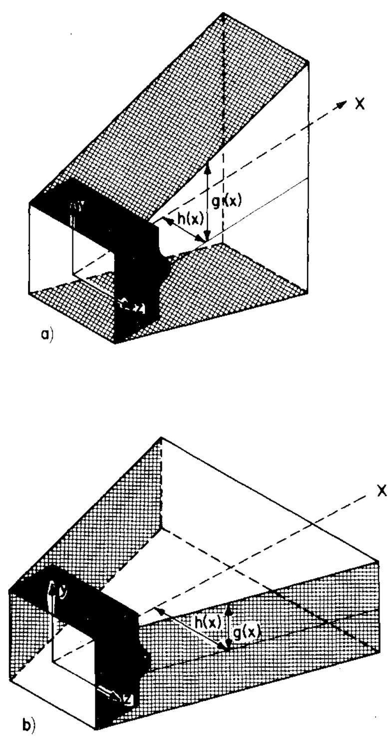

Fig. 1 a) Square inlet diffuser with diffusion in the plane of the velocity profile, b) square inlet diffuser with diffusion normal to the plane of the velocity profile.

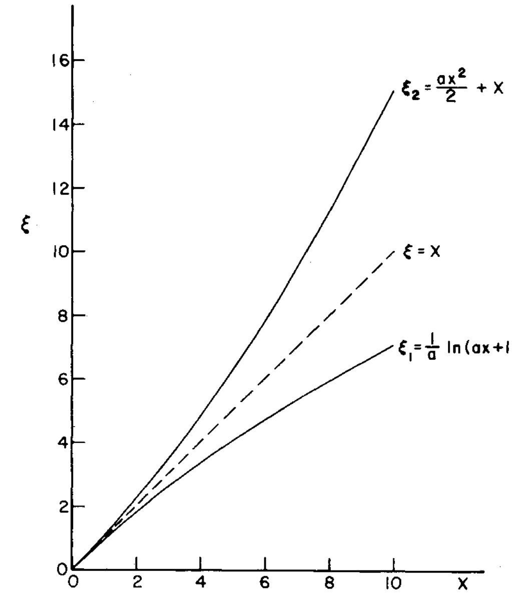

Fig. 2 Effect of diffusion on the streamwise coordinate transformations.

Integrating over the quadrant area $A(x)$

$$\frac{1}{A}\frac{\partial}{\partial x}\int_A u' \, dy \, dz + \frac{d}{dx}\left(\frac{1}{A}\right) + A\frac{dp}{dx} = \frac{1}{R}\int_o^h dz \left.\frac{\partial u'}{\partial y}\right|_o^g \tag{6}$$

In diffuser analyses, it is not uncommon for the core flow to be treated separately from the boundary-layer flow on the diffuser walls. Thus the core flow has an inviscid boundary condition[4] and this, along with the condition of symmetry on the diffuser centerline, means $\partial u'/\partial y|_o^g = 0$ in Eq. (6). The integral term on the lefthand side is zero from Eq. (4). Then

$$\frac{d}{dx}\left(\frac{1}{A}\right) + A\frac{dp}{dx} = 0 \tag{7}$$

or Eq. (3) simplifies to

$$\frac{1}{A}\frac{\partial u'}{\partial x} = \frac{1}{R}\frac{\partial^2 u'}{\partial y^2} \tag{8}$$

Let $\eta = \dfrac{y}{g(x)}$ and $\xi = \displaystyle\int_o^x \left(\frac{h}{g}\right) ds$ (9)

then Eq. (7) becomes

$$(\partial u'/\partial \xi) = (1/R)(\partial^2 u'/\partial \eta^2) \tag{10}$$

with $(\partial u'/\partial \eta) = 0$ at $\eta = 0, 1$ and $u' = F(\eta)$ at $\xi = 0$ (11)

The solution to Eq. (10) with boundary conditions (11) is,

$$u'(\xi_1, \eta) = 2\sum_{n=1}^{\infty} e^{(-n^2\pi^2/R)\xi}\left[\int_o^1 F(\eta)\cos n\pi\eta \, d\eta\right]\cos n\pi\eta \tag{12}$$

III. Effect of Diffusion Direction on the Streamwise Coordinate Transformation

In order to illustrate the effect of direction on the diffusion process, consider the simplest case of a straight walled diffuser. Then, the equation of the expanding walls for diffusion in the plane of the velocity profile, Fig. 1a, is $g(x) = ax + 1$ where a is the slope of the expanding walls. The transformed streamwise coordinate is

$$\xi_1 = \int_o^x \frac{1}{as+1} ds = \frac{1}{a} ln(ax+1) \tag{13}$$

For diffusion in a direction normal to the plane of the velocity profile, Fig. 1b, the equation of the expanding walls (for the same area ratio between exit and inlet) is the same as above, $h(x) = ax + 1$. Then the transformed streamwise coordinate is

$$\xi_2 = \int_o^x (as+1)\, ds = \frac{ax^2}{2} + x \tag{14}$$

Consider two diffusers of equal length but different sets of diverging walls, as illustrated in Figs. 1a and 1b. What is the difference between their exit velocity profiles? The simplest way to examine this question is to compare the two streamwise coordinate transformations as shown in Fig. 2. The two transformed coordinates are plotted vs nondimensionalized physical distance downstream for the case of the wall slope $a = 0.1$. Then at any x position downstream, the corresponding value ξ is directly related to the amount of mixing having taken place up to that streamwise position. Thus, it may be seen from the figure that diffusion normal to the plane of the velocity profile always leads to improved mixing in contrast to diffusion in the plane of the velocity profile, since $\xi_2 > \xi_1$ everywhere.

The difference in the degree of mixing indicated above can be substantial. Defining a variable D as the difference between the transformed coordinates at any x position divided by that x position, the value of D is 19% at an x location of two. The difference D grows very rapidly, reaching values of 28% and 44% at x locations of 3 and 5, respectively.

Some insight into the problem of concurrent mixing and diffusion may be obtained by noting that for no diffusion the transformation of the streamwise coordinate degenerates to $\xi = x$. (It should be kept in mind that the walls are assumed inviscid). This situation is depicted in Fig. 2 by the dashed line. Then it can be clearly seen from the figure that diffusion in the plane of the velocity profile (ξ_1) inhibits the mixing process while diffusion normal to the plane of the velocity profile (ξ_2) enhances the mixing process.

A physical explanation of this process may be the following. For diffusion in the plane of the velocity profile, the diffusion process tends to accentuate the nonuniformity of the velocity profile, while the mixing process tends to reduce the nonuniformity. Thus the two processes oppose each other. For the case of diffusion normal to the plane of the velocity profile, the mixing tends to smooth out the nonuniformities while the diffusion process works in a different direction. Therefore, the two processes do not oppose each other.

IV. Example

As a simple example, consider the deviation from uniformity of the velocity profile at the entrance to the diffuser to be $F(\eta) = b\cos\pi\eta$. The coefficient of the Fourier series solution, Eq. (12), is then

$$b\int_o^1 \cos\pi\eta\cos n\pi\eta \, d\eta = \begin{cases} b/2 & n=1 \\ 0 & n\neq 1 \end{cases} \tag{15}$$

and the solution for u' reduces to

$$u'(\xi, \eta) = b\, e^{(-\pi^2/R)\xi}\cos\pi\eta \tag{16}$$

Thus the shape of the velocity remains qualitatively unchanged while the deviation of the velocity from uniform flow decays as the exponential

$$e^{-\pi^2\xi/R}$$

Now suppose $a = 0.1$, $R = 100$, and the diffuser length is $x_e = 4$ (here R may be viewed as the inverse of an eddy viscosity term). Then the transformed streamwise coordinate is (from Fig. 2) $\xi_1 = 3.36$ for diffusion in the plane of the profile and $\xi_2 = 4.80$ for diffusion normal to the plane. The ratio of the magnitudes of u' at the exit of the diffuser in the two cases is

$$\frac{e^{(-\pi^2/R)\xi_1}}{e^{(-\pi^2/R)\xi_2}} = 1.153$$

Thus the mixing rate has been improved by approximately 15% by diffusing the flow normal to the plane of the velocity profile rather than in the plane.

V. Conclusion

The foregoing analysis indicates the importance of diffusion direction on the linearized mixing process within the diffuser. The same effect may be of even greater importance in the nonlinear case and should be examined both theoretically and experimentally.

References

[1] Quinn, B., "Compact Ejector Thrust Augmentation," *Journal of Aircraft*, Vol. 10, Aug. 1973, pp. 481–486.

[2] Pai, S. I. and Hsieh, T. Y., "Linearized Theory of Three-Dimensional, Jet Mixing With and Without Walls," *Journal of Basic Engineering*, Vol. 92, March 1970, pp. 93–100.

[3] Pai, S. I. and Hsieh, T. Y., "Three-Dimensional Laminar Jet Mixing," *Physics of Fluids*, Vol. 12, April 1969, pp. 936–938.

[4] Wolf, S. and Johnston, J. P., "Effect of Nonuniform Inlet Velocity Profiles on Flow Regimes and Performance in Two Dimensional Diffusers," *Journal of Basic Engineering*, Vol. 91, Sept. 1969, pp. 462–474.

Rocket Plumes in the Thermosphere

JAMES STARK DRAPER* AND FRITZ BIEN†
Aerodyne Research, Inc., Burlington, Mass.
AND
ROBERT E. HUFFMAN‡ AND DUANE E. PAULSEN§
Air Force Cambridge Research Laboratories, Bedford, Mass.

Nomenclature

F = missile thrust
$\bar{L}$ = $[F/q_\infty]^{1/2}$ = hypersonic plume scale
$Kn_{\bar{L}}$ = $\lambda_\infty/\bar{L}$ = plume Knudsen number
q_∞ = $\rho_\infty u_\infty^2/2$ = freestream dynamic pressure
u_∞ = freestream speed
λ_∞ = freestream mean freepath
ρ_∞ = freestream density

Introduction

To provide correct freestream conditions for study of aerodynamic phenomena in the thermosphere (that part of the atmosphere above 90 km altitude), the time-varying properties of the thermosphere must be determined. By maintaining a fixed exospheric temperature of about 1500K as characterizing a "nominal" thermosphere, the standard atmosphere[1] does not allow for these constantly changing thermospheric property altitude profiles. One type of aerodynamic flow frequently studied in the thermosphere is that of rocket exhaust plume. The large property variations away from "nominal" thermospheric conditions can be used to explain the results of recent observations of a vehicle under rocket power in the thermosphere. It will be shown that to understand these observations, the state of the thermosphere must be established through knowledge of the simultaneous solar flux at 10.7 cm and the geomagnetic planetary index, the geographical subpoint and the local solar time, and the day of the year.

Received August 12, 1974; revision received December 5, 1974. This work is a part of Project Chaser, and was monitored by the Air Force Cambridge Research Laboratories under Contract F04611-73-C-0048. The Contract Monitor was D. E. Paulsen. Particular thanks for assistance are given to E. Powers of Aerodyne Research, Inc.

Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Supersonic and Hypersonic Flow; Atmospheric, Space, and Oceanographic Sciences.

* Principal Research Scientist. Member AIAA.
† Research Scientist.
‡ Supervisory Research Chemist.
§ Research Chemist.

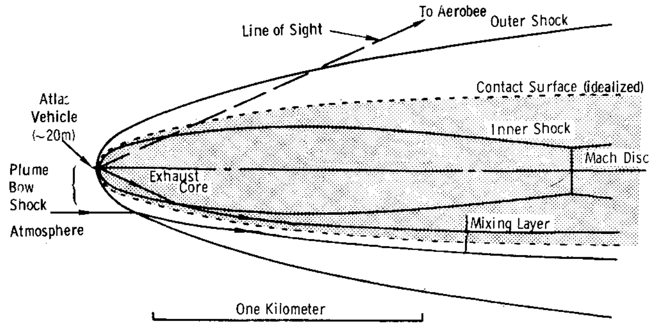

Fig. 1 Schematic of Atlas exhaust plume in the lower thermosphere at an altitude of 100 km showing a representative line of sight to the Aerobee Instrument Package.

Field Measurements

Recently, the Air Force Cambridge Research Laboratories (AFCRL) conducted Project Chaser, a program for the measurement of rocket plumes. Observations of an Atlas-103F vehicle launched at Vandenberg Air Force Base were made on one flight. The AFCRL instrument package, carried by an Aerobee 170 sounding rocket that followed the Atlas vehicle, contained a spatially resolving, ultraviolet photometer. This photometer viewed the Atlas rocket exhaust plume roughly from behind, as shown in Fig. 1. The photometer data has been analyzed to

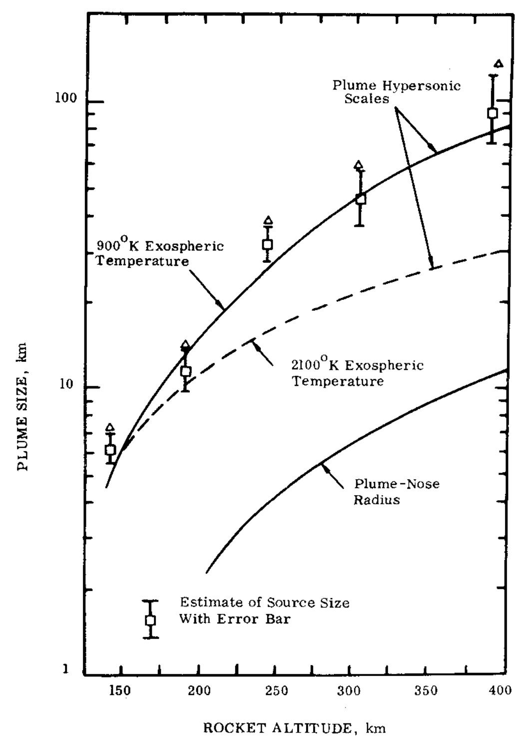

Fig. 2 The size of the observed radiation source vs Atlas altitude. As explained in the text the radiation source was initially assumed to be located at the vehicle, as shown by △. Subsequent analysis shows that the radiation source is probably closer to the plume Mach disk, as shown by the □, to which the error bars have been attached. A comparison is made of the size of the radiation source with the theoretical rate of growth of: 1) the plume mixing layer in "hot" (2100K) and "cold" (900K) exospheric temperature thermospheres; and 2) the growth of the plume bow shock in a 900K exospheric temperature thermosphere.